Minisymposium 11 - Geometrische Analysis

Does finite knot energy imply differentiability?

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In 1991/92 J. O'HARA [1] introduced the family of (j,p)-knot functionals

$$E^{j,p}(\gamma) := \mathscr{L}(\gamma)^{jp-2} \iint_{(\mathbb{R}/(\ell\mathbb{Z}))^2} \left(\frac{1}{|\gamma(s)-\gamma(t)|^j} - \frac{1}{D_\gamma(s,t)^j} \right)^p |\dot\gamma(s)|\,|\dot\gamma(t)|\,\mathrm{d}s\,\mathrm{d}t,$$

where $\gamma \in C^{0,1}(\mathbb{R}/(\ell\mathbb{Z}),\mathbb{R}^3)$ is a curve of length $\mathscr{L}(\gamma)$, the term $D_\gamma(s,t)$ denotes the distance of $\gamma(s)$ and $\gamma(t)$ on γ, and $j,p>0$. The general idea is to produce nice representatives within a given knot class by minimizing these energies, which are self-avoiding iff $jp \geq 2$. In 1994 M. FREEDMAN, Z.-X. HE, and Z. WANG [2] showed for the MÖBIUS *energy* (i. e. $j=2$, $p=1$) that finite energy curves have a local bi-LIPSCHITZ constant arbitrarily close to 1.
Surprisingly there are curves of finite MÖBIUS energy that are not differentiable. In this talk we will present an example of such a curve and ask about the situation for other values of j,p. If we exclude the range of high singularity $\{(j-2)p \geq 1\}$, the answer only depends on the product jp.
This is joint work with SIMON BLATT (RWTH Aachen). [3]

REFERENCES

[1] Jun O'Hara. Family of energy functionals of knots. *Topology Appl.*, 48(2):147–161, 1992.
[2] Michael H. Freedman, Zheng-Xu He, and Zhenghan Wang. Möbius energy of knots and unknots. *Ann. of Math. (2)*, 139(1):1–50, 1994.
[3] Simon Blatt, Philipp Reiter. Does finite knot energy imply differentiability? *Preprints Inst. f. Math. RWTH Aachen*, to appear.